Analysis of a Nondegenerate Two-Photon Giant-Pulse Laser

Introduction

In a recent article\(^1\) to which we shall refer as [S&B], a two-photon laser was proposed and analyzed. This device consists of a cavity resonant at frequency \(v_A\) and containing ions of type \(B\) with an inverted population \(N_B/\nu\) between levels separated by an energy difference \(\hbar\nu_b\) such that \(v_b = 2v_A\); it is necessary that the system not lase at \(v_B\), which criterion can be satisfied by low reflectivity of the cavity at frequency \(v_B\), by strong parasitic absorption in the laser material near frequency \(v_B\), or preferably by a choice of ion such that the transition \(v_B\) is highly forbidden to a single-quantum process. The authors of [S&B] thus show that a certain priming density of photons of frequency \(v_A\) will provoke the simultaneous emission from the inverted population \(N_B\) of pairs of photons \(v_A\) at a rate exceeding the cavity loss, the process diverging until the population inversion is eliminated.

It is the purpose of this communication to show by a very similar analysis that the same system of ions \(N_B\) in a cavity resonant at two frequencies \(v_A\) and \(v_c\), such that \(v_d + v_c = v_B\), may be primed at \(v_A\) with a number of photons small compared with \(N_B\) and will yield two giant pulses simultaneously at frequencies \(v_A\) and \(v_c\). We consider the energy level diagram of Fig. 1. The relaxation of the requirement of [S&B] that \(v_B = 2v_A\) leads to the following: (1) it allows the use of metastable levels \(v_B\) such that \(v_B \gg v_A\) and thus makes available high-intensity laser output in a new short-wavelength range;\(^2\) (2) it allows the production of new laser lines in addition to the amplification of known ones; (3) it eases substantially the problem of designing a system to exhibit the unique fast rise-time characteristics of the multiple-photon laser, which are discussed later in this communication; and (4) it allows the ready production of difference frequencies from the interaction, in a suitable nonlinear medium, of the automatically simultaneous giant pulses.

Equations for the photon population

To the accuracy required for our purposes now, the analysis of [S&B, Eqs. (1) to (13)] makes plausible the following rate equations:

\[
\frac{dS_C}{dt} = B_C S_C S_A N_B - \frac{S_C}{\tau} \tag{1}
\]

\[
\frac{dS_A}{dt} = B_C S_C S_A N_B - \frac{S_A}{\tau} \tag{2}
\]

\[
\frac{dN_B}{dt} = -B_C S_C S_A N_B \tag{3}
\]

in which \(S_A\) and \(S_C\) are the cavity populations of photons of frequency \(v_A\) and \(v_C\) respectively. The cavity decay-time \(\tau\) is assumed common for the two sets of photons, and the two-photon coupling constant \(B_1\) is given in [S&B].

Figure 1 Energy level scheme for the two-photon laser. The inverted population \(N_B\) is prevented from lasing by means of low cavity \(Q\) or by choice of a very long spontaneous lifetime. The cavity has a high \(Q\) at both \(v_B\) and \(v_A\); and the laser will be “primed” with an initial population \(S,(0)\) photons at frequency \(v_A\). (For simplicity we consider only a single mode at \(v_c\) and \(v_A\)).

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\(^1\)S&B

\(^2\)Reference
The behavior of the multi-photon laser is simply treated in three time regimes: I, exponential growth of the minority photon population; II, giant pulse, during which $S_A$ and $S_C$ grow together and $N_B$ approaches 0; and III, decay. We shall first discuss Regimes II and III, and then shall treat the priming requirements of Regime I.

**Regime II: Development of the giant pulse**

In this regime $S_A \approx S_C \equiv S$, and the terms in $1/\tau$ are negligible. Thus Eqs. (1) and (2) become

$$\frac{dS}{dt} \approx B_1 S^3 N_B \approx -\frac{dN_B}{dt}$$

which indicates a maximum logarithmic growth rate

$$\frac{1}{S} \frac{dS}{dt} \bigg|_{\text{max}} \approx B_1 S N_B(0) \approx B_1 N_B(0)^2,$$

which may far exceed the logarithmic growth rate of a conventional giant pulse laser. Eq. (4) would yield the solution

$$\left(1/S_s\right) - \left(1/S\right) = B_1 N_B t,$$

showing that the giant pulse total growth time is on the order of $1/[S_0 B_1 N_B(0)]$. Equations (3) and (4) are readily solved together in the form

$$\frac{d\sigma}{dT} = \sigma^2 (1 - \sigma)$$

[which $\sigma = S/N_B(0)$ and $T = B_1 N_B^2(0)t$, and in which we have noted that $N_B(t) \approx N_B(0) - S(t)$ according to Eq. (4)] with the indefinite integral

$$\frac{-1}{\sigma} - \ln \left(\frac{1}{\sigma} - 1\right) = T$$

giving rise to the plot of Fig. 2.

From the parameters given in [S&B], $N_B(0) = 2 \times 10^{15}$ and $B_1 = 3.6 \times 10^{-25}$ sec$^{-1}$, we find the time unit $[B_1 N_B^2(0)]^{-1}$ to be $0.7 \times 10^{-12}$ sec, and from Fig. 2 we see that

$$\frac{1}{S} \frac{dS}{dt} \bigg|_{\text{max}} \approx \frac{B_1 N_B^2(0)}{4} \approx 3 \times 10^{13}$$. sec$^{-1}$.

Such enormous rates of change of population justify the neglect of the $1/\tau$ terms in this growth regime. Incidentally, they also cast some doubt on the quantitative validity of this model of a homogeneous cavity population in the presence of a growth rate corresponding to $\sim 1$ mm travel of light!

**Regime III: Decay**

As has been shown, the growth of the photon population and the deexcitation of all of the ion inverted population occurs in a time much less than the cavity decay-time $\tau$. Thus Regime III is simply an exponential decay, $S_A \approx S_B \approx N_B e^{-t/\tau}$.

**Regime I: Priming conditions**

- **Method I**

  The priming criteria require some special discussion. We assume that the laser is primed with a substantial population $S_A(0)$ of photons $\nu_A$. The condition for growth of the $\nu_A$ population, according to Eq. (1), is

  $$B_1 S_A(0) N_B \geq \frac{1}{\tau}$$

  (7)

  or

  $$S_A > S_0 = \frac{1}{B_1 N_B \tau},$$

  (8)

  which is identical with [S&B, Eq. (17)] except for a trivial factor of 2. Thus if Eq. (7) is well satisfied, $S_C$ will grow exponentially with a time constant $1/B_1 S_A N_B(0)$ until $S_C$ is no longer small compared to $S_A$. More precisely, one scheme for priming is to fill the cavity to a level $S_A(0)$ satisfying Eq. (8), and to allow the $\nu_A$ population to decay freely while the $\nu_C$ population grows. The condition that $S_C \approx S_A$ before $S_A$ decays below the critical level, Eq. (8), is thus readily seen to be

  $$S_A(0) \geq \ln \left(\frac{S_0}{1}\right)$$

  (9)

  (considering the initial “spontaneous” emission from the $[N_B \rightarrow S_A(0)]$ system into the $\nu_C$ mode as being induced by the zero-point energy of the vacuum). Thus Method I requires an initial priming photon density about 30 times as great as is necessary for the degenerate two-photon laser of [S&B].

- **Method II**

  An alternative to Method I is to supply priming photons
Figure 3  Course of events in the nondegenerate two-photon laser. This hypothetical system of \( N_A(0) = 2 \times 10^6 \) inverted \( B \) ions was primed with a total of \( \sim 10^8 \) \( A \) photons, a priming energy of \( 10^{-8} \) joule.

\( r_A \) over a period of several cavity decay-times. We can thus calculate the total number of photons \( r_A \) and the corresponding supply time required for reaching Regime II. The result is that the number is a minimum for instantaneous supply as in Method I and is

\[
S_{\text{min}} \approx S_0 \ln S_0, \tag{10}
\]

but that the total number of priming photons required does not increase by much so long as one pumps well over the threshold, Eq. (8). Thus, \( 2S_0 \ln S_0 \) expresses the number of \( r_A \) photons required if one maintains a photon level \( 2S_0 \) in the cavity for a time \( \tau \ln S_0 \approx 10^{-7} \) sec, using the parameters of [S&B]. Normal laser spikes exceed \( 10^{-7} \) seconds in duration, so that the nondegenerate two-photon laser can be primed by the same photon source that would be adequate for the degenerate case.

Figure 3 shows the course of the various populations as a function of time, using Method I for priming, whereas Figure 2 shows the steep region of the pulse on a time scale expanded \( \sim 10^4 \) times.

Discussion

We have analyzed briefly the predicted performance of a nondegenerate two-photon laser. The triggering requirements would be eased by a higher cavity \( Q \) for the priming photons \( r_A \). They seem stiff but not impossible.

The very high logarithmic-growth-rate of the two-quantum laser deserves some comment. Normal \( Q \)-switched lasers are limited in growth rate by the condition that the cavity in the low-\( Q \) status be stable against the exponential growth of population in the resonant modes. Thus, if the cavity time constant is switchable between \( \tau/\tau \) and \( \tau \), the above condition requires the build-up time in the absence of loss to be longer than \( \tau/\tau \). For pink ruby, the \( Q \)-switched rise time has been shown to be about \( 2 \times 10^{-9} \) sec, about three orders of magnitude larger than the rise time calculated above for the two-photon laser. It remains for experiment to demonstrate the magnitude of improvement actually attainable by the two-photon technique.

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References and footnotes

2. The priming requirement increases by less than a factor of two for \( r_C/3 \leq A \leq 3 \), so that the approximate analysis will be made here for \( r_C = r_A \).
3. Strictly speaking, unless \( S_C \gg 1 \), Eq. (1) should be written \( dS_C/dt = B_S S_A N_B (S_C + 1) - S_C / \tau \), i.e., \( dS_C/dt = B_S S_A N_B - S_C / \tau + B_S S_A N_B \). The last term represents spontaneous emission and is included in the above analysis by starting with \( S_C = 1 \). Equation (9) is obtained as follows: In the priming phase of Method I we have

\[
S_A(t) = S_A(0)e^{(-t/\tau)}, \tag{9a}
\]

and

\[
dS_C = \frac{S_A(t) S_C}{S_0} - \frac{S_C}{\tau} = \frac{S_C}{\tau} \left[ \frac{S_A(0)}{S_0} e^{-t/\tau} - 1 \right], \tag{9b}
\]

which integrates directly to

\[
\ln \left[ \frac{S_C(t)}{S_C(0)} \right] = \frac{S_A(0)}{S_0} \left( 1 - e^{-t/\tau} \right) - \frac{t}{\tau}, \tag{9c}
\]

which for \( S_C(0) = 1 \) and \( S_A(0)/S_0 \gg 1 \), gives Eq. (9).


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